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Data Analysis and Advanced Modeling Techniques

Editor

Sadi ELASAN

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PREFACE

Today, where the information age is rapidly evolving, statistical science, numerical data analysis and advanced modeling techniques play an indispensable role in scientific research and academic studies. This book, "Numerical Data Analysis and Advanced Modeling Techniques", is a scientific resource created with the contributions of leading experts on the subject. This book aims to provide readers with a strong academic foundation by addressing digital data analysis and advanced modeling within a scientific framework. Each chapter takes an interdisciplinary approach and expands on advanced statistical methods.

Each author provides in-depth knowledge in his/her field of research, giving readers a broad perspective. Each chapter is supported by real-world application examples and analysis, enriched both theoretically and practically. From this book; Academicians working in scientific research, students and professionals interested in advanced data analysis and modeling techniques can benefit from it. The book will be a valuable resource for anyone wishing to gain a more in-depth understanding of numerical data analysis and advanced modelling.

Information is power, and an important way to obtain information is to analyze data with statistical models. This book aims to provide a strong academic foundation in statistics, numerical data analysis and advanced modeling. We hope that scientists will benefit from this numerical data analysis and modeling subjects in scientific research and academic studies.

Editor

Assoc. Prof. Dr. Sadi ELASAN

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CHAPTER I

A Study On Software Reliability With Kalman Filter

Levent ÖZBEK[1]

Introduction

The successful operation of a software is one of the important factors that determine software reliability. The definition of software reliability is the probability of error-free operation over a period of time. Standard models have an important place in software reliability modeling. These models ignore observation noise and its impact on software reliability in accurate evaluation. This research created a reliability model using time series and state-space models method. This model is converted into a state space model. Noise was reduced using the Kalman filter. The established model was applied to the resulting data about the Linux operating system kernel. The results obtained using these data revealed that this method performed well.

[1] Associate Professor, Ankara Üniversity, Department of Statistics, Ankara, Türkiye. ORCID ID: 0000-0003-1018-3114

For the goodness of the model, the evaluation was made by calculating absolute and relative error measures.

Adaptive Kalman filter (AKF)

Consider the state-space model given below.

$$x_{t+1} = F_t x_t + G_t w_t \tag{1}$$

$$y_t = H_t x_t + v_t \tag{2}$$

here x_t is an state, y_t is an observation vector, F_t is an system and H_t is an observation matrix The covariance matrices w_t and v_t are defined by $w_t \sim N(0, Q_t)$ and $v_t \sim N(0, R_t)$. The KF equations are

$$\hat{x}_{t|t-1} = F_{t-1}\hat{x}_{t-1} \tag{3}$$

$$P_{t|t-1} = F_{t-1}P_{t-1|t-1}F^{'}_{t-1} + G_{t-1}Q_{t-1}G^{'}_{t-1} \tag{4}$$

$$K_t = P_{t|t-1}H^{'}_t(H_t P_{t|t-1}H^{'}_t + R_t)^{-1} \tag{5}$$

$$P_{t|t} = [I - K_t H_t]P_{t|t-1} \tag{6}$$

$$\hat{x}_t = \hat{x}_{t|t-1} + K_t(y_t - H_t\hat{x}_{t|t-1}) \tag{7}$$

KF works properly when the values of the covariance matrices in the model are known. For various reasons, these matrices are unknown in real applications. Various adaptive methods have been studied in the literature to ensure proper functioning of KF [5-7]. In the article [6], the forgetting factor for KF was proposed. This adaptive form of KF was used in this study.

$$P_{t|t-1} = \alpha\left(F_{t-1}P_{t-1|t-1}F^{'}_{t-1} + G_{t-1}Q_{t-1}G^{'}_{t-1}\right) \tag{8}$$

The software model

A time series studied can be decomposed into two components: trend and cycle, such that

$$y_t = T_t + C_t \tag{9}$$

here y_t is the software data. T_t and C_t denote the trend and cycle at time t. The component C_t is assumed to follow a second-order autoregressive proceses (AR(2)). The AR(2) parameters are assumed to random walks. In formulas,

$$C_t = a_t C_{t-1} + b_t C_{t-2} + \varepsilon_{1,t} \tag{10}$$

here $\varepsilon_{1,t}$ is iid sequence. The trend component is specified as a random walk with drift given by

$$T_t = \mu_{t-1} + T_{t-1} + \varepsilon_{2,t} \tag{11}$$

$$\mu_t = \mu_{t-1} + \varepsilon_{3,t} \tag{12}$$

here $\varepsilon_{2,t}$ and $\varepsilon_{3,t}$ are iid sequence. If the state-space form of equations (9-12) are written as

$$x_{t+1} = \begin{bmatrix} a_t & b_t & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} x_t + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} w_t \tag{13}$$

then

$$y_t = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix} + v_t \tag{14}$$

where $x_t = \begin{bmatrix} C_t & C_{t-1} & T_t & \mu_t \end{bmatrix}'$ is the state and a_t and b_t are parameters. Let $F_t(\theta)$ is a function of the parameter $\theta = \begin{bmatrix} a_t & b_t \end{bmatrix}'$. Let θ is the random walk processes,

$$\theta_{t+1} = \theta_t + \zeta_t \tag{15}$$

where ζ_t is iid sequence with $Cov(\zeta_t) = S_t$. The system can be re-formulated as the nonlinear model:

$$\begin{bmatrix} x_{t+1} \\ \theta_{t+1} \end{bmatrix} = \begin{bmatrix} F_t(\theta_t)x_t \\ \theta_t \end{bmatrix} + \begin{bmatrix} w_t \\ \zeta_t \end{bmatrix} \tag{16}$$

$$y_t = [H_t \quad 0] \begin{bmatrix} x_t \\ \theta_t \end{bmatrix} + v_t \tag{17}$$

The Adaptive Extended KF (AEKF) can be applied to estimate the state vector [8-10]. Let us the initial state and the covariance as

$$\begin{bmatrix} \hat{x}_0 \\ \hat{\theta}_0 \end{bmatrix} = \begin{bmatrix} E(x_0) \\ E(\theta_0) \end{bmatrix} \tag{18}$$

$$P_0 = \begin{bmatrix} Cov(x_0) & 0 \\ 0 & S_0 \end{bmatrix} \tag{19}$$

then the AEKF equations are

$$\begin{bmatrix} \hat{x}_{t|t-1} \\ \hat{\theta}_{t|t-1} \end{bmatrix} = \begin{bmatrix} F_{t-1}(\hat{\theta}_{t-1})\hat{x}_{t-1} \\ \hat{\theta}_{t-1} \end{bmatrix} \tag{20}$$

$$P_{t|t-1} = \alpha \begin{bmatrix} F_{t-1}(\hat{\theta}_{t-1}) & \frac{d}{d\theta}\left(F_{t-1}(\hat{\theta}_{t-1})\right)\hat{x}_{t-1} \\ 0 & I \end{bmatrix} \times P_{t-1} \begin{bmatrix} F_{t-1}(\hat{\theta}_{t-1}) & \frac{d}{d\theta}\left(F_{t-1}(\hat{\theta}_{t-1})\right)\hat{x}_{t-1} \\ 0 & I \end{bmatrix}^{'} +$$

$$+\alpha \begin{bmatrix} G_{t-1}Q_{t-1}G_{t-1}^{'} & 0 \\ 0 & S_{t-1} \end{bmatrix} \tag{21}$$

$$\begin{bmatrix} \hat{x}_t \\ \hat{\theta}_t \end{bmatrix} = \begin{bmatrix} \hat{x}_{t|t-1} \\ \hat{\theta}_{t|t-1} \end{bmatrix} + K_t \left[y_t - H_t \hat{x}_{t|t-1} \right] \tag{22}$$

$$P_t = \left(I - K_t \left[H_t \;\; 0\right]\right) P_{t|t-1} \tag{23}$$

$$K_t = P_{t|t-1}\left[H_t \;\; 0\right]' \left[\left[H_t \;\; 0\right] P_{t|t-1} \left[H_t \;\; 0\right]' + R_t \right]^{-1} \tag{24}$$

Application and results

First data are taken from the articles [11]. This date set is given Table-1. The real and estimated values obtained with our proposed method are given in Fig.1. When Fig.1 is examined, the estimation results are very close to the real values. Relative error values are shown in Fig.2. When Fig.2 is examined, it is seen that the relative error values are quite small and oscillate around zero.

***Table-1** Linux: Cumulative number of faults per million lines of code (Month)*

Time	Faults	Time	Faults	Time	Faults
1	3.	26	36.	51	62
2	6.	27	36.	52	62
3	11.	28	37.	53	65
4	13.	29	37.	54	65
5	15.	30	37.	55	65
6	17.	31	39.	56	68
7	19.	32	39.	57	68
8	19.	33	39.	58	68
9	21.	34	42	59	74
10	21.	35	42	60	74
11	23.	36	45	61	81
12	23.	37	45	62	81
13	26.	38	45	63	81
14	26.	39	48	64	87
15	26.	40	48	65	87
16	30.	41	52	66	87
17	30.	42	52	67	94
18	30.	43	52	68	94
19	32.	44	56	69	94
20	32.	45	56	70	101
21	34.	46	56	71	101
22	34.	47	59	72	101
23	35.	48	59	73	110
24	35.	49	59	74	110
25	35	50	62	75	118

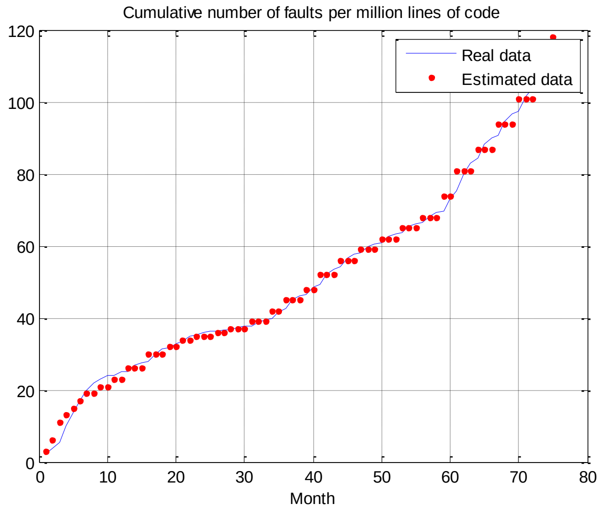

Figure-1: Real data and Estimation

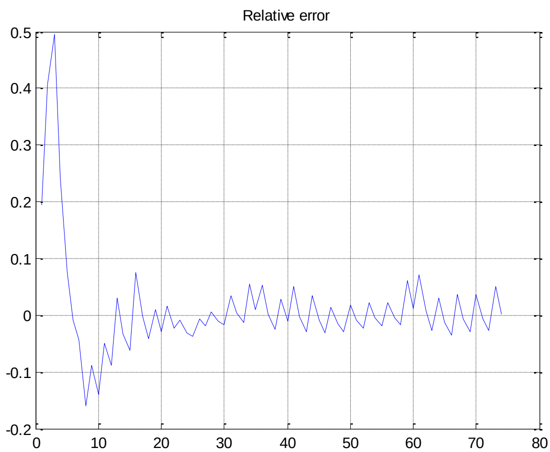

Figure-2: Relative Error

In order to see how the estimation method would give a result on another data set, the second data set in the reference given in [12] was used. This date set is given Table-2. The real and estimated values obtained with our proposed method are given in Fig.3. When

Fig.3 is examined, the estimation results are very close to the real values. Relative error values are shown in Fig.4. When Fig.4 is examined, it is seen that the relative error values are quite small and oscillate around zero.

Table 2 *Linux: Cumulative number of faults per million lines of code (Month)*

Time	Faults	Time	Faults	Time	Faults
1	0.5098	26	57.625	51	89.375
2	10.197	27	57.625	52	89.375
3	18.584	28	58.961	53	92.625
4	21.928	29	58.961	54	92.625
5	25.240	30	58.961	55	92.625
6	28.541	31	61.597	56	95.812
7	31.832	32	61.597	57	95.812
8	31.832	33	61.597	58	95.812
9	35.122	34	61.597	59	102.043
10	35.122	35	65.467	60	102.043
11	38.361	36	69.228	61	108.960
12	38.361	37	69.228	62	108.960
13	43.002	38	69.228	63	108.960
14	43.002	39	72.930	64	114.448
15	43.002	40	72.930	65	114.448
16	49.040	41	77.781	66	114.448
17	49.040	42	77.781	67	120.502
18	49.040	43	77.781	68	120.502
19	51.991	44	82.487	69	120.502
20	51.991	45	82.487	70	126.350
21	54.853	46	82.487	71	126.350
22	54.853	47	85.989	72	126.350
23	54.853	48	85.989	73	133.532
24	56.253	49	85.989	74	133.532
25	56.253	50	89.375	75	139.727

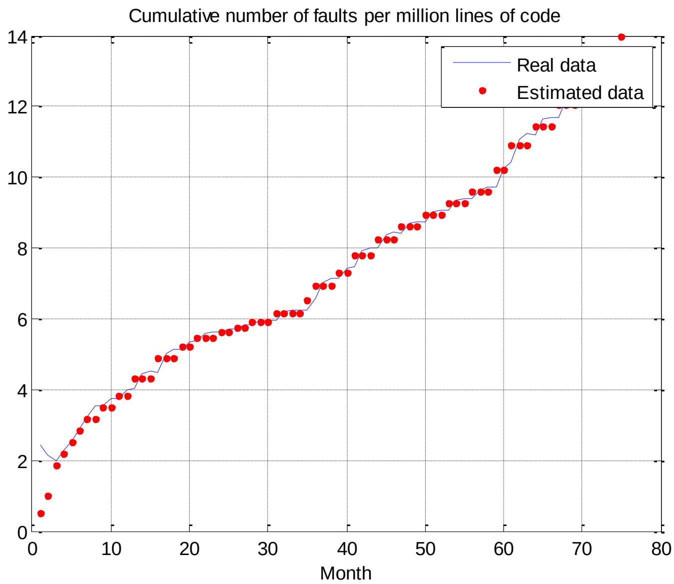

Figure-3: Real data and Estimation

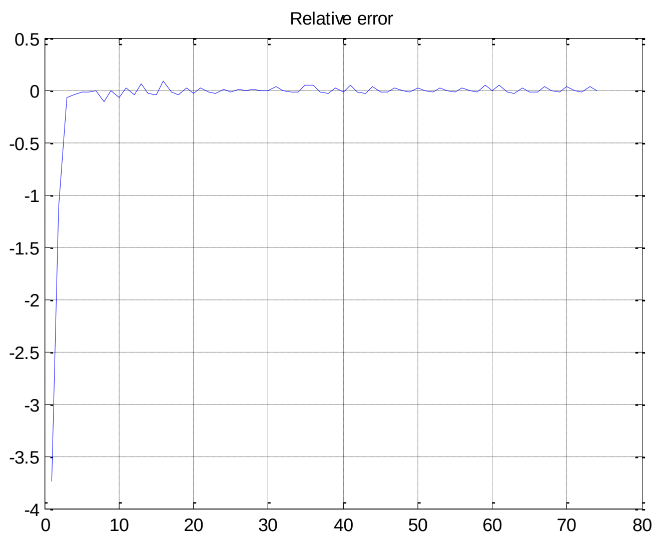

Figure-4: Relative Error

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CHAPTER II

A Study On The Number π And Geometric Distribution

Levent ÖZBEK[1]

Introduction

The number π is one of the most elegant flowers in the mathematical garden. It has been a flower that mathematicians and other scientists have smelled with curiosity and interest for hundreds of years since Archimedes (Öztürk & Özbek, 2015). This number has many features: It is a transcendent number, that is, it is a number that cannot be the root of a polynomial whose coefficients are integers. The proof of this was made by Ferdinand von Lindemann in 1882. His proof was based on two centuries of important mathematical contributions. The $e^{i\pi}+1=0$ equation, which we

[1] Associate Professor, Ankara Üniversity, Department of Statistics, Ankara, Türkiye. ORCID ID: 0000-0003-1018-3114

encounter in the mathematics literature and whose aesthetic properties are often mentioned by mathematicians, is quite interesting in that it contains $e, i, \pi, 1, ve\ 0$ numbers, which are the most important constant numbers of mathematics. Although many different methods are used to calculate π, today convergent infinite series, multiplications and sequential recurrence relations are used (Borwein, 2000).

For thousands of years, people have been trying to calculate more decimal places of π, and it is a matter of curiosity how these decimal places are distributed. Where does this interest in π come from? What other properties of π are ready to be discovered other than those known so far? The most elegant flower of the mathematical garden stands there and perhaps waits like a lover ready to offer its infinite features. In almost all mathematics books, especially those written for people who are interested in mathematics, the properties of π are mentioned. It's really interesting to see how π is used differently in geometry, probability, differential and integral calculations (Özbek, 2018).

Why would anyone want to calculate the value of π to billions of digits, as is done today with supercomputers? What is the source of this interest in the decimal places of π? This is used to measure the capabilities of supercomputers' hardware and software. Computational methods can lead to new ideas and concepts. Doesn't π have any order or pattern? Does it contain an endless variety of patterns? Are some numbers in π more common? Aren't these numbers randomly distributed? Perhaps the interest and admiration that mathematicians have felt for π throughout the centuries can be compared to the strong desires and emotions that drive mountain climbers to climb higher and higher.

In fact, the answers to all these questions have not been given clearly yet. A new research article about π is published every day. As long as human curiosity and passion continues, the desire to find a new aesthetic direction in π seems to continue forever.

This study was carried out to show that the digits of π have a geometric distribution.

Statistical Properties of The Digits of π

Studies to date on the decimal digits of π have shown that these numbers pass all statistical (random) tests (Dodge, 1996; Jaditz, 2000; Lange, 1999; Osler, 1999; Ganz, 2014; Bailey, & Borwein, 2012; Ganz, 2017; Bailey, & Borwein, 2017). It should also be noted that a new statistical test may be developed and these numbers may fail this test. Although it seems that there is no order in these decimal places (which has not been found to date), researchers continue their studies under the assumption that there may be an order. Many methods have been developed and are still being developed to calculate the digits of π.

Geometric Distribution in π Number

Let's give an example on the number 1 to determine that the digits in π are a random sequence. Under the assumption that the numbers are random and uniform distributed, the arrival of the numbers 1 will occur in a geometric distribution with probability $p = 0.1$.

Let X be the number of non-1 digits between the next number 1 after a 1 appears anywhere.

$X=0$ means the next number is also 1. Thus, $P(X=0)=p=0.1$.

$X=1$ means the next number is different from 1 and the second is 1. Thus, $P(X=1)=qp=0.09$.

$X=k$ means the next k numbers are different from 1 and the last number is 1. Thus, $P(X=k)=q^k p$

Thus, to test whether the 1's are random and uniform, the numbers between the 1's in the given sequence (within the digits of π) are counted. How many 0s, how many ones, how many binary ones, and so on. Counting the intervals, the table is created as follows (Table 1).

Table 1: Chi square table

X	0	1	2	3	4	5	6	7	8	9	10	11	12	13	...	39	40	>40
Arrivals																		
Expectation																		

Arrivals up to a certain place in the number π, for example, if the sum of the arrivals row within *30000* digits is *n*, this number corresponds to the number of experiments in the hypothesis test.

$nq^k p$ values are written in the expectation line. To perform the chi-square goodness-of-fit test, the following test statistic is calculated.

$$\chi^2 = \sum_{i=0}^{41} \frac{(arrivel_i - \exp ectation_i)^2}{\exp ectation_i}$$

By comparing this calculated value with the Chi-square table value, it can be said whether the digits of the π number comply with the geometric distribution. All of these operations can be done in the same way for the numbers *0,1,2,3,...,9*.

When the digits of the number π are examined with the idea explained above, the values observed in *30000* digits for the numbers *0,1,2,3,...,9* are found as given in Table-2. The graphs of the values given in Table-1 are given in Figure 1- Figure 10, respectively. As can be seen from the figures, the observed values resemble a geometric distribution. Expected values calculated for 0 are given in Figure-11. Since there will be the same values for *1,2,…*9, their shapes are not given.

Chi-square values calculated using observed and expected values are given in Table-3. Considering the *alpha=0.05* significance level and the number of classes as *43*, the Chi-square table value is *59.304*, so it can be said that the delays for all numbers *0,1,2,…9* comply with the geometric distribution.

Table 2: Observed values for numbers 0,1,2,….9

0	1	2	3	4	5	6	7	8	9
0 297	0 305	0 293	0 273	0 275	0 309	0 296	0 288	0 296	0 308
1 280	1 285	1 248	1 278	1 284	1 264	1 235	1 245	1 259	1 260
2 257	2 267	2 207	2 232	2 253	2 271	2 252	2 259	2 243	2 263
3 206	3 259	3 232	3 218	3 233	3 215	3 232	3 239	3 216	3 221
4 231	4 190	4 179	4 218	4 233	4 214	4 212	4 201	4 190	4 220
5 177	5 175	5 183	5 171	5 182	5 167	5 180	5 158	5 153	5 156
6 162	6 165	6 146	6 172	6 168	6 185	6 167	6 158	6 150	6 159
7 137	7 123	7 136	7 116	7 152	7 159	7 142	7 143	7 135	7 147
8 115	8 150	8 126	8 111	8 128	8 127	8 133	8 111	8 135	8 133
9 119	9 113	9 110	9 115	9 127	9 100	9 117	9 116	9 109	9 104
10 95	10 107	10 97	10 99	10 104	10 89	10 112	10 120	10 111	10 105
11 94	11 73	11 95	11 95	11 103	11 123	11 104	11 96	11 109	11 84
12 77	12 88	12 65	12 83	12 83	12 81	12 82	12 75	12 103	12 89
13 75	13 79	13 65	13 84	13 80	13 59	13 88	13 70	13 71	13 98
14 78	14 58	14 63	14 77	14 64	14 68	14 65	14 69	14 62	14 57
15 56	15 60	15 57	15 69	15 56	15 57	15 49	15 60	15 71	15 69
16 48	16 42	16 66	16 64	16 66	16 71	16 51	16 62	16 63	16 53
17 55	17 57	17 46	17 46	17 48	17 44	17 51	17 45	17 51	17 45
18 37	18 35	18 50	18 50	18 41	18 49	18 46	18 51	18 48	18 45
19 43	19 37	19 52	19 37	19 35	19 33	19 33	19 39	19 48	19 31
20 34	20 42	20 34	20 35	20 37	20 45	20 41	20 33	20 36	20 34
21 41	21 26	21 35	21 38	21 29	21 25	21 36	21 29	21 32	21 31
22 26	22 36	22 31	22 33	22 20	22 38	22 29	22 21	22 28	22 28
23 19	23 20	23 27	23 30	23 27	23 26	23 31	23 38	23 24	23 19
24 17	24 25	24 19	24 29	24 16	24 29	24 17	24 20	24 18	24 33
25 23	25 25	25 25	25 23	25 19	25 13	25 27	25 15	25 20	25 19
26 25	26 23	26 28	26 25	26 19	26 24	26 19	26 16	26 16	26 16
27 15	27 15	27 15	27 12	27 16	27 17	27 13	27 25	27 16	27 19
28 21	28 16	28 17	28 10	28 14	28 12	28 5	28 23	28 9	28 17
29 16	29 23	29 22	29 17	29 14	29 12	29 17	29 16	29 9	29 21
30 9	30 7	30 14	30 12	30 5	30 8	30 12	30 9	30 9	30 12
31 10	31 13	31 6	31 4	31 10	31 11	31 16	31 11	31 17	31 13
32 8	32 7	32 8	32 16	32 16	32 9	32 7	32 17	32 13	32 10
33 4	33 14	33 10	33 7	33 13	33 12	33 15	33 14	33 8	33 14
34 6	34 11	34 5	34 15	34 16	34 10	34 11	34 6	34 12	34 13
35 10	35 4	35 9	35 6	35 6	35 13	35 3	35 8	35 8	35 3

36 3	36 6	36 4	36 7	36 2	36 5	36 7	36 6	36 6	36 3
37 8	37 5	37 7	37 6	37 9	37 3	37 6	37 6	37 11	37 6
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64 2	64 1	64 0	64 0	64 0	64 0	64 0	64 0	64 0	64 0
65 3	65 0	65 0	65 0	65 0	65 1	65 0	65 0	65 0	65 0
66 0	66 1	66 0	66 0	66 0	66 0	66 1	66 0	66 0	66 0
67 1	67 0	67 1	67 0	67 0	67 0	67 0	67 0	67 0	67 0
68 3	68 0	68 0	68 1	68 0	68 0	68 0	68 0	68 0	68 0
69 0	69 0	69 0	69 1	69 0	69 0	69 0	69 0	69 0	69 0
70 0	70 0	70 1	70 0	70 1	70 0	70 1	70 0	70 0	70 2
71 0	71 0	71 0	71 0	71 0	71 1	71 0	71 0	71 0	71 0
72 0	72 0	72 1	72 1	72 0	72 0	72 0	72 0	72 0	72 0
73 0	73 0	73 0	73 0	73 0	73 0	73 0	73 0	73 0	73 0
74 0	74 0	74 0	74 0	74 0	74 0	74 0	74 1	74 0	74 0
75 0	75 1	75 0	75 0	75 1	75 0	75 0	75 0	75 0	75 1
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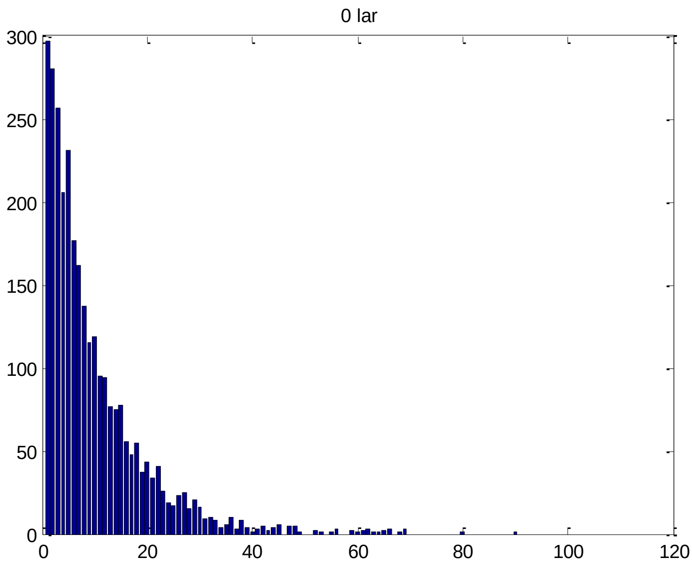

Figure-1: Delays for 0

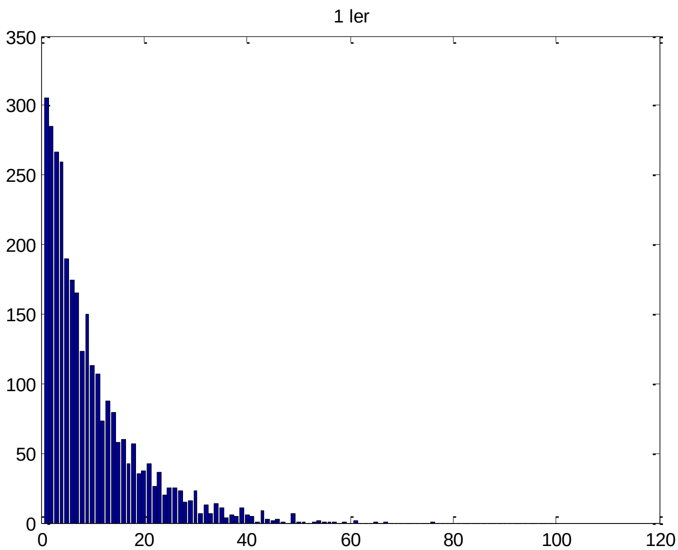

Figure-2: Delays for 1

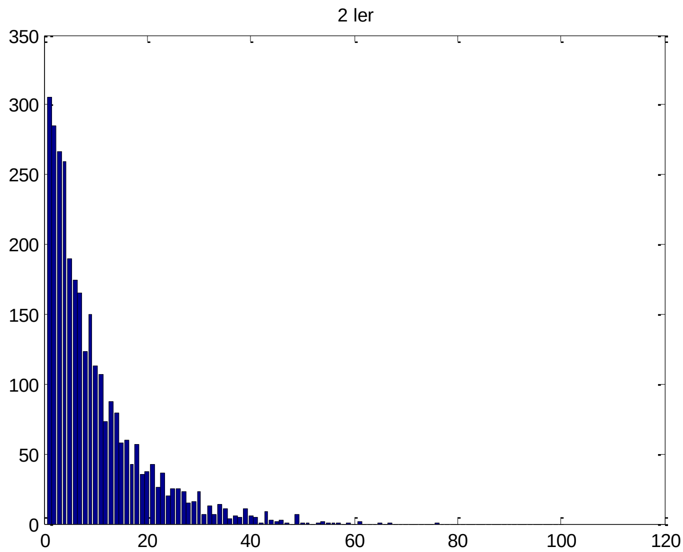

Figure-3: Delays for 2

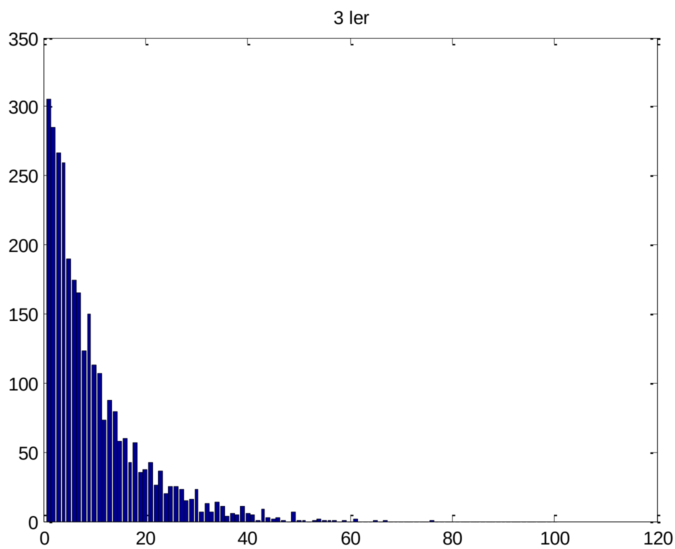

Figure-4: Delays for 3

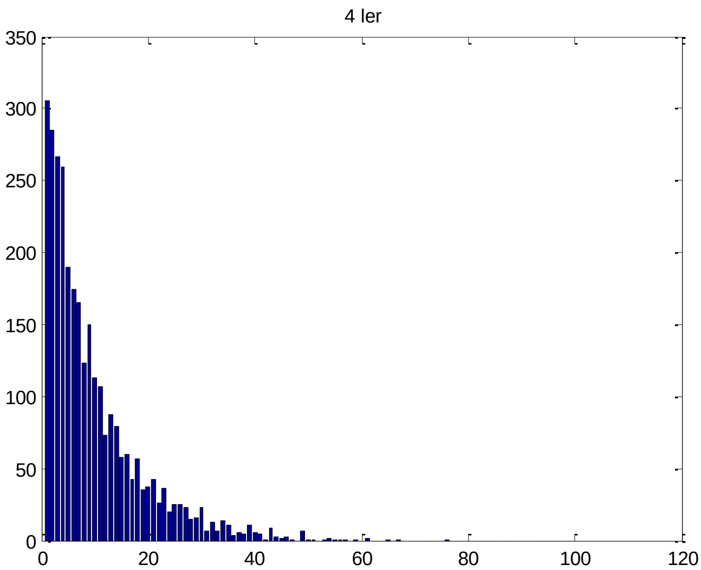

Figure-5: Delays for 4

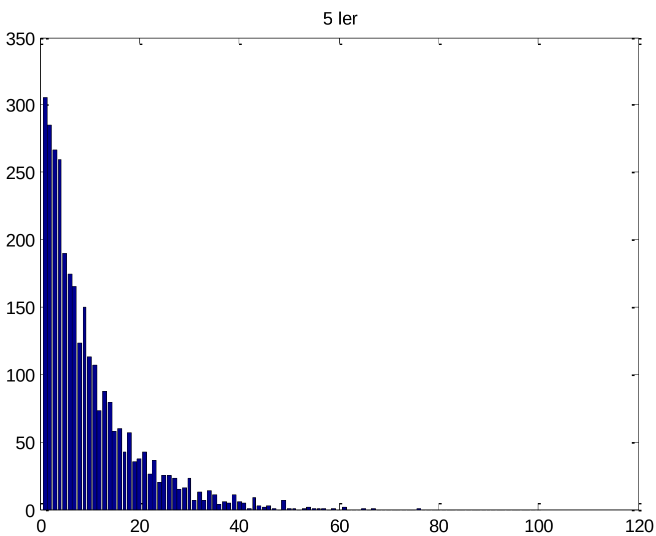

Figure-6: Delays for 5

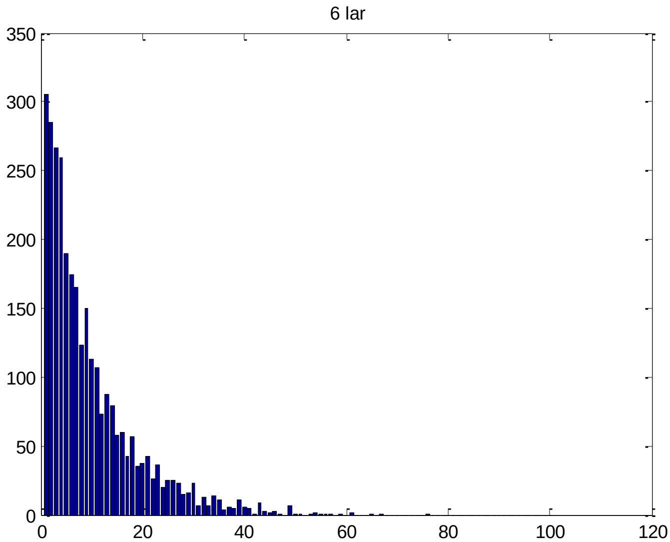

Figure-7: Delays for 6

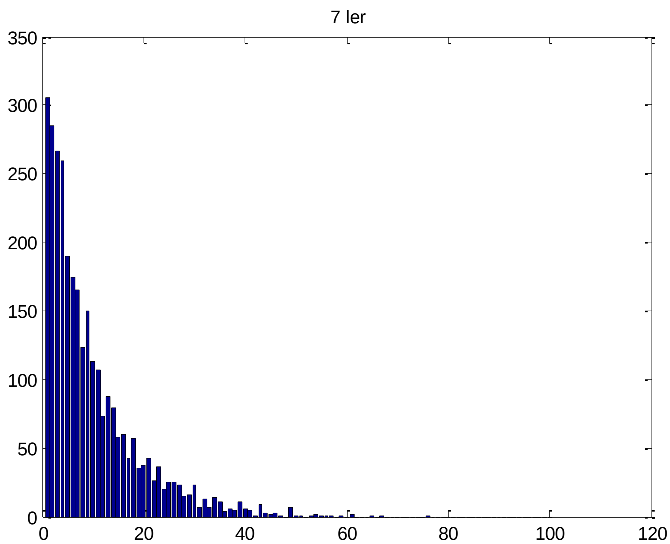

Figure-8: Delays for 7

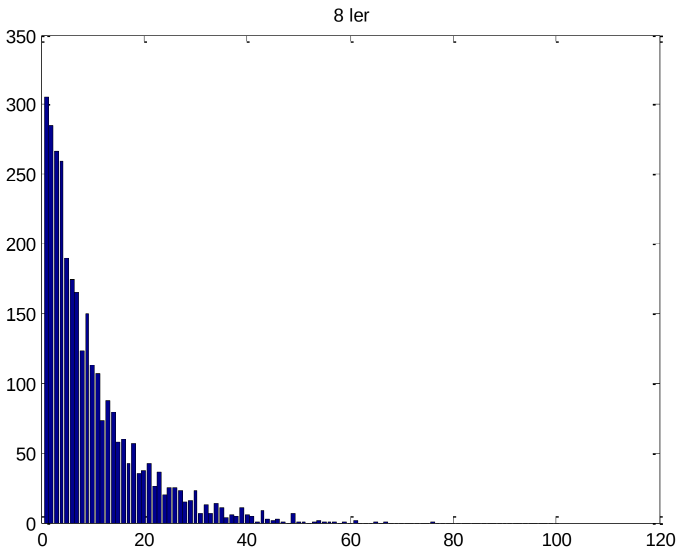

Figure-9: Delays for 8

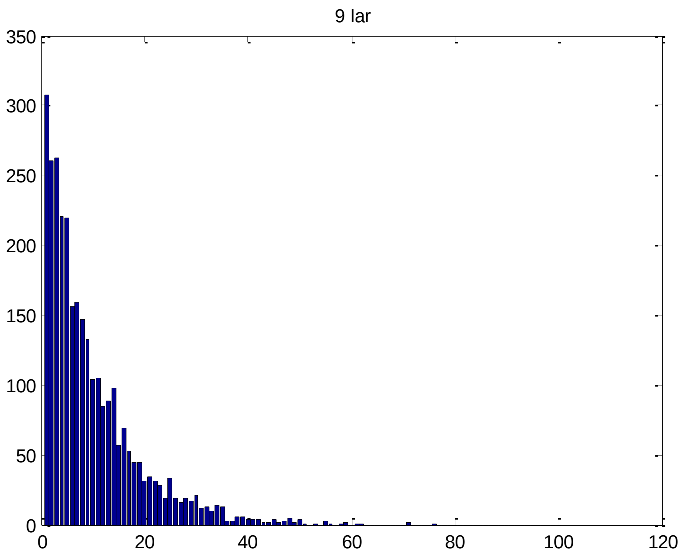

Figure-10: Delays for 9

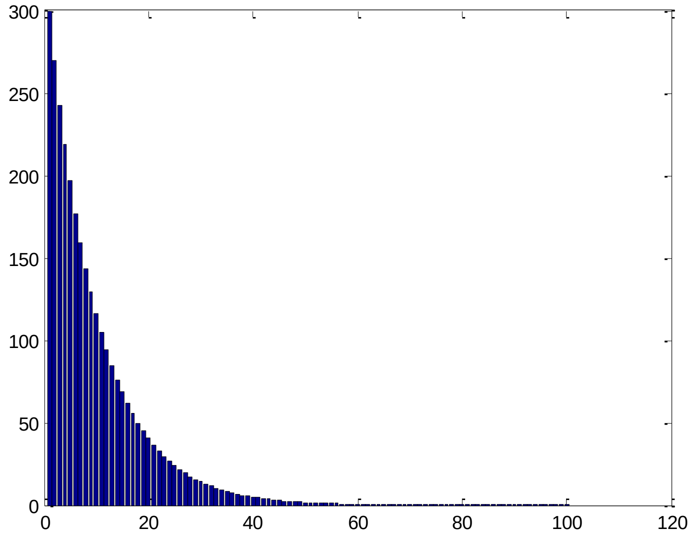

Figure-11: Expected values for 0

Table-3: Calculated Chi-square values

number	*Chi-square*
0	44,69
1	56,77
2	45,52
3	47,69
4	46,37
5	58,40
6	40,17
7	43,89
8	37,61
9	42,71

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Appendix: Digits of π

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CHAPTER III

Estimating The Time-Varying Beta Risk with GARCH and GJR-GARCH Models Under Different Distribution Assumptions

Merve PAKER

1. INTRODUCTION

The theory modeling the risk-return relationship between financial assets was first developed by Markowitz [1]. The modern portfolio theory (MPT) includes the variance-covariance model, which suggests that the risk will decrease with the diversification of financial assets in the portfolio. Based on this theory is the Capital Asset Pricing Model (CAPM), which is frequently preferred by investors due to its ease of implementation and flexibility of parameters proposed by Sharpe, Lintner and Mossin [2, 3, 4].

The CAPM includes the beta risk parameter and gives how the financial asset changes according to the market and whether the risk is high. The Linear Market Model (LMM), which is consistent with this model, summarizes beta risk with a stationary or fixed beta parameter. One of the most important assumptions of the model is the linearity between the variables in the model. It has been proven in many studies that this assumption cannot be met [5, 6, 7, 8]. Therefore, The Conditional Capital Asset Pricing Model (C-CAPM) was in which the time-varying beta parameter is used instead of the stationary or fixed beta parameter created by Jagannathan and Wang [9]. That is, the Time-varying Linear Market model (Tv-LMM), which is consistent with this model, summarizes the beta risk with the dynamic or time-varying beta parameter. In the literature, it has been observed that GARCH-type models are frequently preferred for the time-varying beta parameter [10, 11, 12, 13, 14, 15, 16, 17]. In addition, there are some of these studies were carried out for different sectors. [10, 18, 19, 20, 21, 22, 23, 24, 25]

In this paper, the first to address the systematic risk or also known as beta risk of the information, communications, security, investigative activities, defense, office management, office support and other company support activities industry. The beta risk or systematic risk measurement of the information, communications, security, investigative activities, defense, office management, office support and other company support activities sector in Turkey, which is missing in the literature, is discussed. For this purpose, a portfolio was created by taking the daily frequency data of all information, communications, security, investigative activities, defense, office management, office support and other company support activities companies in the BIST National All index covering the period 04.11.2021-18.04.2022. The Conditional Capital Asset Pricing Model (C-CAPM), which allows time-varying beta parameter, was used as the basic model. The time-varying beta parameter is modeled with GARCH-N, GARCH-SN, GARCH-T, GARCH-ST, GJR-GARCH-N, GJR-GARCH-SN, GJR-GARCH-T and GJR-GARCH-ST that are univariate GARCH-type models. Additionally, features and effects on these models are described. First of all, it is aimed to guide the investors who want to invest in this sector and to summarize the different features and effects of the models at the date of research in this sector. in addition, the garch models, which are the volatility models that have become more famous in recent years, and the beta risk, which is systematic risk, have been examined for this sector, which has not been investigated before, researchers have been provided with an idea, and contribute to the literature.

2. METHOD AND MATERIAL

2.1. Financial Models

2.1.1. Capital Asset Pricing Model (CAPM)

In the finance literature, CAPM or Two-Moment CAPM is the most generally prefered model to investigate the systematic risk measure beta risk, in other words, systematic covariance risk or systematic beta, put forward by Sharpe, Lintner and Mossin [2, 3, 4]. This model is defined as equation [7[. The Linear Market Model (LMM) is consistent with CAPM and is the data generation process of CAPM, is defined as equation (1). This model is based on the MPT developed by Markowitz [1]. LMM is the model that allows stable beta risk (β_{im}). It is based on the assumption that the asset returns are normally distributed and the investor's utility function is of second order, that is, the utility can only be expressed with the mean and variance measures [10]. That is why it is called a two-moment model. In the model, the mean criterion expresses the expected return, and the variance criterion expresses the risk.

$$R_{it} - R_{ft} = a_i + \beta_{im} (R_{mt} - R_{ft}) + \varepsilon_{it} \quad (1)$$

$$i = 1, \ldots, n, \quad t = 1, \ldots, T$$

The slope of the model (β_{im}) is the beta coefficient that is defined as the beta risk of the financial asset i.. R_{it} is return on financial asset i. at time t.. R_{ft} is risk-free rate return at time t. and R_{mt} is the return on portfolio at time t.. Here, $R_{mt} - R_{ft}$ *is* excess return on portfolio (R_{mt}) relative to the risk free return over time t.

and $R_{it} - R_{ft}\ is$ excess return on financial asset i. (R_{it}) relative to the risk-free return over time t.. The a_i coefficient is becomes zero when the market is active, the prices in the period of interest are not affected by past prices, and the price change is assumed to be random (random walk theory). In this case, the error terms (ε_{it}) are independent, with constant variance and same distribution, and the coefficient of a_i is assumed to be zero according to the Sharpe-Lintner-Mossin version of CAPM. In this case ε_{it}, financial asset i. are the residuals of $i \neq k$ for $\varepsilon_{it} \sim N(0, \sigma_i^2)$ and $j > 0$ for $E\left(\varepsilon_{it}\varepsilon_{i\,t+j}\right) = 0$ in time of t..

Estimation of β_{im} is, under the assumption $\varepsilon_{it} \sim N(0, \sigma^2)$, defined in equation (2).

$$\hat{\beta}_{im} = \frac{\sum_{t=1}^{T}[(R_{it}^* - \bar{R}_i^*)(R_{mt}^* - \bar{R}_m^*]}{\sum_{t=1}^{T}[(R_{mt}^* - \bar{R}_m^*)^2]} = \frac{Cov(R_i, R_m)}{Var(R_m)} \tag{2}$$

$$R_{mt}^* = R_{mt} - R_{ft} \tag{3}$$

$$R_{it}^* = R_{it} - R_{ft} \tag{4}$$

$$\bar{R}_i^* = \frac{1}{T}\sum_{t=1}^{T} R_{it}^* \tag{5}$$

$$\bar{R}_m^* = \frac{1}{T}\sum_{t=1}^{T} R_{mt}^* \tag{6}$$

where R_{it}^* is the excess return on financial asset i. at time t., R_{mt}^* is the excess return on portfolio at time t., $\bar{R}_i^*$ is the average excess return on financial asset i. on the total time., $\bar{R}_m^*$ is average excess return on portfolio on the total time. $Cov(R_i, R_m)$ is the covariance between the return on financial asset i. and on portfolio, $Var(R_m)$ is variance on the portfolio.

CAPM is defined in equation (7).

$$E(R_i) - R_f = \beta_{im}[E(R_m) - R_f] \qquad (7)$$

$$i = 1, \ldots, n$$

where R_i, R_m, R_f are return on financial asset i., portfolio and risk-free rate, recpectively. $E(R_i)$ and $E(R_m)$ are expective return on financial asset i. and portfolio, respectively. $E(R_i) - R_f$ is expected excess return on financial asset i. relative to the risk-free return. $E(R_m) - R_f$ is expected excess return on portfolio relative to the risk-free return. β_{im} is investment risk and market risk of financial asset i..

2.1.2. Conditional Capital Asset Pricing Model (C-CAPM)

While including the constant or stable beta risk parameter (β_{im}) into the model that the CAPM, including the time-varying or dynamic beta risk parameter (β_{imt}) into the model that the Conditional Capital Asset Pricing Model (C-CAPM) [18]. The Time-varying Linear Market Model (Tv-LMM) is is consistent with C-CAPM and allows time-varying beta risk (β_{imt}).

Tv-LMM is defined in equation (8).

$$R_{it} - R_{ft} = a_i + \beta_{imt}(R_{mt} - R_{ft}) + \varepsilon_{it} \qquad (8)$$

$$i = 1, \ldots, n, \quad t = 1, \ldots, T$$

In this model, beta risk (β_{imt}) is calculated based on time. β_{imt} is beta risk of financial asset i.. at time t. and defined in equation (10). C-CAPM is defined in equation (9).

$$E(R_{it}) - R_{ft} = \beta_{imt}[E(R_{mt}) - R_{ft}] \qquad (9)$$

$i = 1, \ldots, n, \quad t = 1, \ldots, T$

$$\beta_{imt} = \frac{Cov(R_{it}, R_{mt})}{Var(R_{mt})} \tag{10}$$

where $Cov(R_{it}, R_{mt})$ is the covariance between the return on financial asset i. and on portfolio at time t., $Var(R_{mt})$ is variance on the portfolio at time t. So, the variance in this model should be calculated as time varying. Where, autoregressive conditional variance is used for the time-varying variance.

2.2. Statistical Models

2.2.1. Generalized Autoregressive Conditional Variance-GARCH

The discrete-time stochastic process which information of daily returns rt is expressed as [26]

$$r_t = m_t + \varepsilon_t \tag{11}$$

$$\varepsilon_t = h_t Z_{t,} \quad Z_t \sim i.i.d.N(0,1) \tag{12}$$

$$h_t = Var(r_t | F_{t-1}) = Var(x_t | F_{t-1}) \tag{13}$$

where ε_t is an independent and identically distributed (i.i.d) process with a zero mean and one standard deviation, and F_{t-1} denotes the information available at time $t-1$. The aforementioned equation forms the foundation of all volatility model. This volatility model one of GARCH (p,q) model introduced by Bollerslev [27] is given by,

$$h_t^2 = w + \sum_{i=1}^{p} \psi_i Y_{t-i}^2 + \sum_{j=1}^{q} \theta_j \sigma_{t-j}^2 \tag{14}$$

$$\mathrm{t} = \min(\mathrm{p}, \mathrm{q}) + 1, \ldots, \mathrm{n}$$

where $\omega > 0,\ \psi_i \geq 0$ and $\theta_j \geq 0$ constraints were defined by Nelson and Cao [28] so that the conditional variance model parameters are positive at every t. Another constraint in the model is the constraint of stationarity of covariance. For this, the condition $\sum_{i=1}^{p} \psi_i + \sum_{j=1}^{q} \theta_j < 1$ must be met [10]. The coefficient sums in the GARCH model give the persistence of volatility in the face of a shock/news. If the sum is equal to 1, the GARCH model transforms into IGARCH (the integrated generalized autoregressive conditional variance) model.

Owing to the features of financial time series like extreme kurtosis, volatility clustering, leverage effect, studies on GARCH-type models have been carried out and developed in the past and today.

2.2.2. Glosten-Jagannathan-Runkle GARCH- GJR-GARCH

One of these most popular models is GJR-GARCH model created by Glosten-Jagannathan-Runkle [29], which models the leverage effect. The leverage effect here was defined by Black [30] and it is defined as the asymmetric reaction of volatility to positive and negative shocks in series. GJR-GARCH (p, q) model is given by,

$$h_t^2 = w + \sum_{i=1}^{p} (\psi_i Y_{t-i}^2 - \zeta_i I_{t-i} Y_{t-i}^2) + \sum_{j=1}^{q} \theta_j \sigma_{t-j}^2 \tag{15}$$

where ζi is the leverage term that t. value 1 if the value is negative or zero while t. value 0 if the value is positive an indicator variable $It{-}i$ is defined.

2.2.3. Normal Distribution

The two-parameter distribution, called the Normal Distribution described by two moments which the mean and variance. The Normal Distrubition description by ND~ (μ, σ^2) has probability density function (PDF) given by,

$$f(x) = \frac{e^{\frac{0.5(x-\mu)^2}{\sigma^2}}}{\sigma\sqrt{2\pi}} \quad (16)$$

A mean filtration or whitening process, the residuals ε, standardized by σ yield the Standard Normal Distrubition given by,

$$f\left(\frac{x-\mu}{\sigma}\right) = \frac{1}{\sigma} f(z) = \frac{1}{\sigma}\left(\frac{e^{-0.5z^2}}{\sqrt{2\pi}}\right) \quad (17)$$

To obtain the conditional likelihood of the GARCH-N process at each point in time is given by,

$$LL_t(\psi) = -0.5\left(Tln(2\pi) + \sum_{t=1}^{T} lnh_t^2 + \sum_{t=1}^{T} \varepsilon_t^2\right) \quad (18)$$

where ψ = (m, ω, ψ_i , $\boldsymbol{\zeta_i}$) denotes the parameter vector of the GARCH-N model.

2.2.4. Skew-Normal Distrubition

The Skew-Normal (SN) Distrubition description. SN Distrubition has probability density function (PDF) is given by,

$$f(x;\lambda) = 2\emptyset(x)\phi(x\lambda), \qquad x, \lambda \epsilon IR \quad (19)$$

where φ (·) and Φ (·) are the probability density function (PDF) and cumulative distrubition function (CDF) of Standard Normal Distribution, respectively. λ is the skewness term that SN distribution is left-skewed for λ < 0, otherwise, it is right skewed and when λ = 0, the SN distribution reduces to standard normal distribution.

Standard Skew-Normal Distrubition given by,

$$f(\varepsilon;\lambda) = 2\sigma\emptyset(\varepsilon\sigma + \mu)\phi(\varepsilon\sigma\lambda + \mu\lambda), \quad x, \lambda \epsilon IR \tag{20}$$

In this formula μ and σ are given by,

$$\mu = \sqrt{{}^{2}/_{\pi}}\,\delta \tag{21}$$

$$\sigma = 1 - {}^{2}/_{\pi}\,\delta^2 \tag{22}$$

To obtain the conditional likelihood of the GARCH-SN process at each point in time is given by,

$$LL_t(\psi) = Tln(2\pi) + \sum_{t=1}^{T} \ln\left(\phi(\varepsilon_t\sigma + \mu)\right) + \sum_{t=1}^{T} \ln\left(\Phi(\varepsilon_t\sigma\lambda + \mu\lambda)\right) - \frac{1}{2}\sum_{t=1}^{T} \ln\left(h_t^2\right) \tag{23}$$

where ψ = (m, ω, ψ_i , $\boldsymbol{\zeta_i}$, λ) denotes the parameter vector of the GARCH-SN model.

2.2.5. Student-t Distrubition

The three-parameter distribution, called the Student-t Distribution described by three parameters which the mean, variance and shape (ν). The Student-t Distribution has probability density function (PDF) given by,

$$f(x) = \frac{\Gamma\left(\frac{\upsilon+1}{2}\right)}{\sqrt{\beta\upsilon\pi}\,\Gamma\left(\frac{\upsilon}{2}\right)}\left(1 + \frac{(x-\alpha)^2}{\beta\upsilon}\right)^{-\left(\frac{\upsilon+1}{2}\right)} \tag{24}$$

where α, β, and ν are the location, scale and shape parameters respectively, the ν parameter is tail-thickness and Γ is the Gamma function. Substituting (ν−2)/ ν in (24) obtained Standard Student-t Distrubition.

Standard Student-t Distrubition given by,

$$f\left(\frac{x-\mu}{\sigma}\right) = \frac{1}{\sigma} f(z) \tag{25}$$

$$= \frac{1}{\sigma}\frac{\Gamma\left(\frac{\upsilon+1}{2}\right)}{\sqrt{(\upsilon-2)\pi}\,\Gamma\left(\frac{\upsilon}{2}\right)}\left(1 + \frac{z^2}{\upsilon-2}\right)^{-\left(\frac{\upsilon+1}{2}\right)}$$

To obtain the conditional likelihood of the GARCH-Student-t (GARCH-T) process at each point in time is given by,

$$LL_t(\psi) = T\left(ln\Gamma\left(\frac{\upsilon+1}{2}\right) - ln\Gamma\left(\frac{\upsilon}{2}\right) - \frac{1}{2}\ln\left(\pi(\upsilon-2)\right) - \frac{1}{2}\sum_{t=1}^{T}(lnh_t^2 + (1+\upsilon)ln\left(1+\frac{\varepsilon_t^2}{\upsilon-2}\right)\right) \quad (26)$$

where ψ = (m, ω, ψ_i , $\boldsymbol{\zeta_i}, \upsilon$) denotes the parameter vector of the GARCH-T model.

2.2.6. Skew Student-t Distrubition

The Skew Student-t (ST) Distrubition has probability density function (PDF) is given by,

$$f(x;\lambda,\upsilon) = 2t(x;\upsilon)T\left(\sqrt{\frac{1+\upsilon}{x^2+\upsilon}}\lambda x;\upsilon+1\right), x\epsilon IR \quad (27)$$

where t(·) and T (·) are the probability density function (PDF) and cumulative distrubition function (CDF) of Student-t distribution, respectively. λ and υ are the skewness and tail-thickness parameters, respectively. ST distribution is left-skewed for $\lambda < 0$, otherwise, it is right skewed and when $\lambda = 0$, the ST distribution reduces to Student-t distribution.

Standard Skew Student-t Distrubition given by [],

$$f(\varepsilon;\lambda,\upsilon) = 2\sigma t\left((\varepsilon\sigma+\mu);\upsilon\right)T \quad (28)$$

$$\left(\sqrt{\frac{1+\upsilon}{(\varepsilon\sigma+\mu)^2+\upsilon}}\right)\lambda(\varepsilon\sigma+\mu);\upsilon+1,\ \ \upsilon > 2$$

In this formula μ and σ are given by,

$$\mu = \frac{\left(\frac{\upsilon}{\pi}\right)^{\frac{1}{2}}\Gamma\left(\frac{\upsilon-1}{2}\right)}{\Gamma\left(\frac{\upsilon}{2}\right)}\frac{\lambda}{\sqrt{1+\lambda^2}} \tag{29}$$

$$\sigma = \left(\frac{\nu}{\nu-2} - \mu^2\right) \tag{30}$$

To obtain the conditional likelihood of the GARCH-ST process at each point in time is given by,

$$LL_t(\psi) = Tln(2) + Tln\sigma + \sum_{t=1}^{T} \ln\left(t(\varepsilon_t\sigma+\mu);\upsilon\right) + \sum_{t=1}^{T}\ln\left(\mathrm{T}\left(\sqrt{\frac{1+\upsilon}{(\varepsilon\sigma+\mu)^2+\upsilon}}\,\lambda(\varepsilon\sigma+\mu);\ \upsilon+1\right)\right) - \frac{1}{2}\sum_{t=1}^{T}\ln\left(h_t^2\right) \tag{31}$$

where ψ = (m, ω, ψ_i , $\boldsymbol{\zeta_i}$, λ, υ) denotes the parameter vector of the GARCH-ST model.

3. RESULT

The research data of this paper covers the dates of 4 November 2021 to 18 April 2021. In this date range, a portfolio was created by taking daily frequency data of all quarrying and mining companies in the BIST National All index. The 3-month Turkish Lira Reference Interest Rate (TRLIBOR) is preferred used and risk free rate data from http://www.trlibor.org/veriler.aspx.

The abbreviations of the research data are given in Table 1 that XUTUM and all the information, communications, security, investigative activities, defense, office management, office support and other company support activities companies on the XUTUM.

The daily returns for all mining and quarrying companies in the BIST National All index and the BIST National All market portfolio were obtained by the first difference of the logarithm of closing price of Turkish lira.

$$R_{it} = \ln(P_{it}) - \ln(P_{it-1}) \qquad (32)$$

The three-month Turkish Interbank Offered Rate (TRLIBOR) interest rate served in percentage per annum ($TRLIBOR_t$), they can be converted to a daily rate of return as follows.

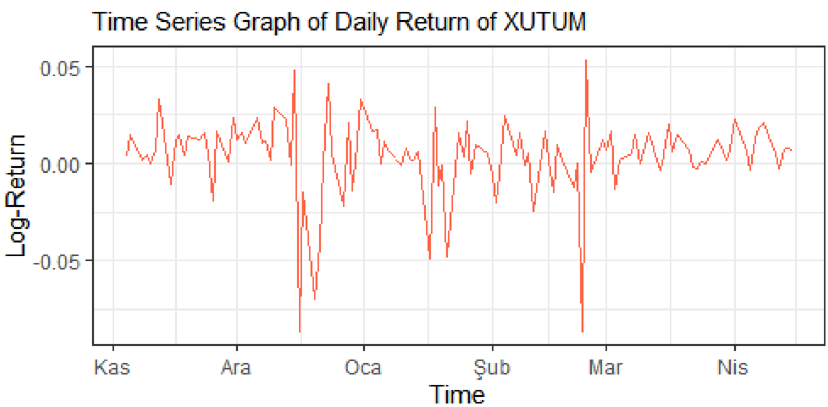
$$R_{ft} = \left(1 + \frac{TRLIBOR_t}{100}\right)^{\frac{1}{252}} - 1 \qquad (33)$$

The abbreviations of the research data are given in Table 1 and Figure 1 gives the time series plots of close price of the companies on the XUTUM.

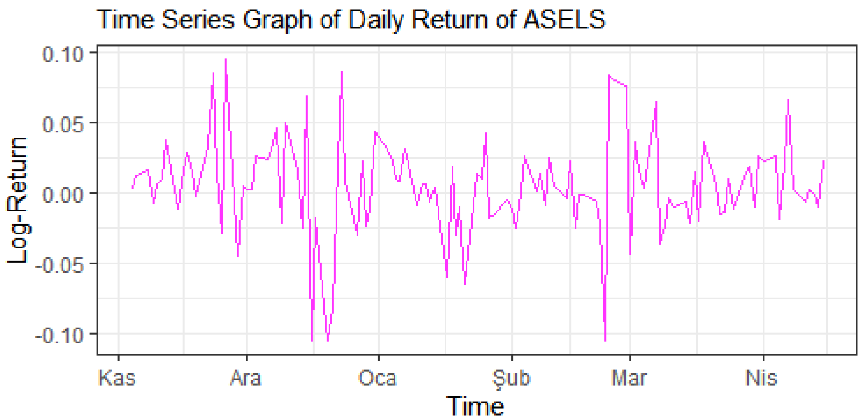
Table 1. Names and Codes of the Country's Stocks Exchange and Companies

Codes	Explain of Codes
XUTUM	BIST National All
ASELS	Aselsan Electronic Industry and Trade Inc.
CEOEM	CEO Event Media Inc.
FLAP	Flap Congress Meeting Services, Automotive and Tourism Inc.
IHAAS	Ihlas New Agency Inc.
SNKRN	Senkron Security and Communication Systems Inc.

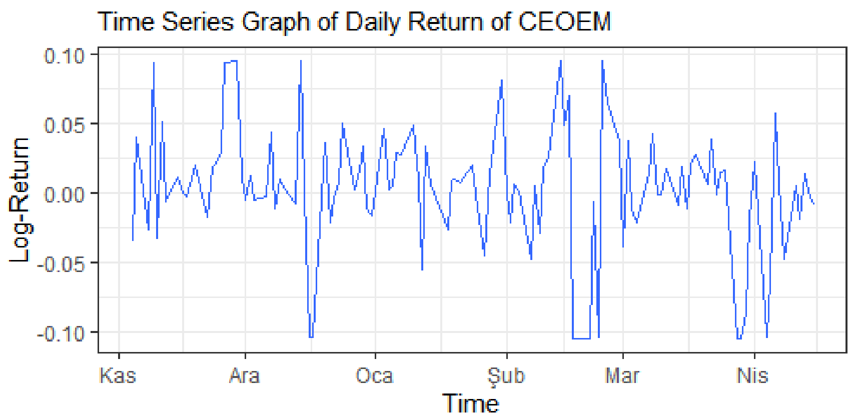
Figure 1 shows the time series graphs of returns on the XUTUM and all these companies, respectively. When the graphs here are examined, it is observed that the trends in the companies and the movements of the companies over time are consistent with the comments given in Table 2. The date 2020 was defined as the COVID-19 global epidemic by the WHO and the effects on the markets consequently of the global economic that is extreme fluctuations, was clearly observed.



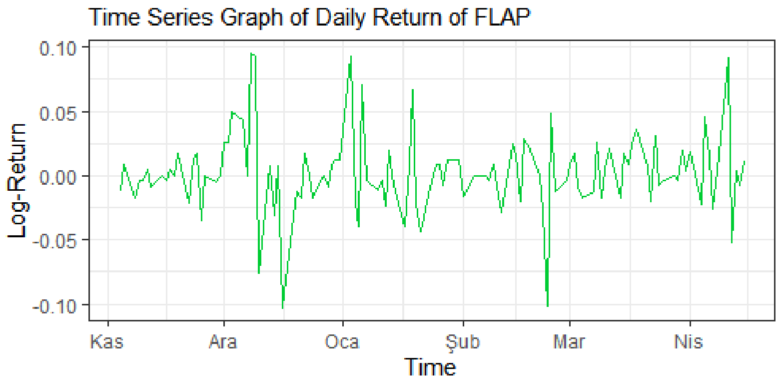
(a)


Time Series Graph of Daily Return of ASELS
Log-Return
0.10
0.05
0.00
-0.05
-0.10
Kas
Ara
Oca
Şub
Mar
Nis
Time

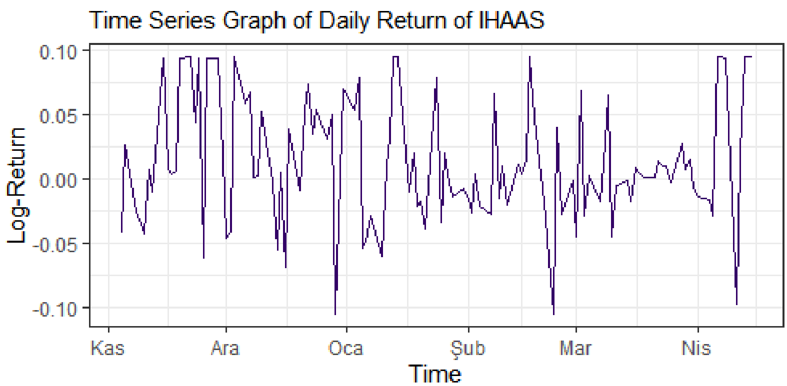
(b)


Time Series Graph of Daily Return of CEOEM
Log-Return
0.10
0.05
0.00
-0.05
-0.10
Kas
Ara
Oca
Şub
Mar
Nis
Time

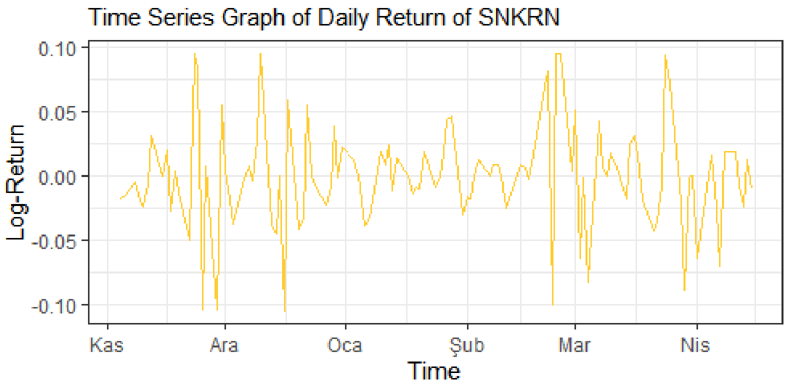
(c)


Time Series Graph of Daily Return of FLAP
Log-Return
0.10
0.05
0.00
-0.05
-0.10
Kas
Ara
Oca
Şub
Mar
Nis
Time

(d)


Time Series Graph of Daily Return of IHAAS
Log-Return
0.10
0.05
0.00
-0.05
-0.10
Kas
Ara
Oca
Şub
Mar
Nis
Time

(e)



(f)

Figure 1. Time Series Plots of Returns: (a) XUTUM; (b) ASELS; (c) CEOEM; (d) FLAP; (e) IHAAS; (f) SNKRN

Table 2 gives descriptive statistics for the daily returns on the XUTUM market portfolio and the 5 global markets in the XUTUM. This table that the mean return on the daily XUTUM is 0.0036, with a standard deviation of about 0.0206. A positive XUTUM average indicates that investors who will invest in XUTUM during the research period will make a profit financially. The range of mean from 0.0036 for XUTUM to 0.0119 for IHAAS, meaning that IHAAS generated greater financial profit on investment than XUTUM on this period. The mean return on all the companies is more than the mean risk-free rate (TRLIBOR), which stand for the minimum return an investor theoretically expects for any investment, recomming that investors would prefer to invest in these sector on this period. The highest standard deviation is that for ASELS (0.3554), while the lowest one is that of XUTUM (0.0206). For this reason, when the standart deviation is accepted as a risk measure, it can be said that the riskiest company is the ASELS. The return distributions of 2 company, show positive skewness that there are frequent small drops and a few excessive increases in returns, while negative skewness means that there are frequent small increases and a few excessive drops in returns. The range of skewness between -1.8676 (XUTUM) and -4.2401 (TRLIBOR). This shows that investment experiences increases and a few excessive drops in terms of investment, while reports frequent small drops and a few excessive increases returns. The return distributions of all the companies except IHAAS are leptokurtic, meaning that the market has fatter tails than the normal distribution (which has kurtosis $\geq$ 3) and more chance of excessive outcomes. The range of kurtosis between 2.4715 (IHAAS) and 36.1487 (TRLIBOR). This shows that IHAAS has less chance of excessive financial losses or profits than the other investments. The normality of XUTUM, risk-free rate (TRLIBOR) and each investment except IHAAS and SNKRN, are also rejected at the 5% significance level using the Jarque-Bera (JB) test which is probable to be owing to skewness and kurtosis observed in Table 2. To test the autocorrelation for the squared returns (proxies for volatilities) of the investment, the XUTUM and the risk-free rate, the Ljung-Box (LB) test is used in this study. According to the LjungBox (LB) test, the null hypothesis of no autocorrelation for the squared returns is not rejected at the 5% significance level for all companies and the XUTUM, and the risk-free rate meaning that there have not autocorrelation for the squared returns. The ARCH effect of each investment is also rejected at the 5% significance level using the ARCH-LM (LM) test. This result show that all companies can modeling of GARCH-type models. As results supply mightly evidence for the estimability of the volatility for the companies, the XUTUM and the risk-free rate (Christoffersen, 2003). Achieved results provide effects such as the principal features of these data are the asymmetry (left-skew and right-skew), positive mean, relatively high volatility, and leptokurtosis (fat tails) over the performance of all models while estimating the time-varying volatility. And these features match the most common features of market studies (Harvey, 1995).

Table 2. Descriptive Statistics of Daily Returns

	Mean	Std. Dev.	Skewness	Kurtosis
XUTUM	0.0036	0.0206	-1.8676	9.4455
ASELS	0.0045	0.3554	-0.3014	4.7982
CEOEM	0.0019	0.0454	-0.5222	3.8276
FLAP	0.0015	0.0318	0.0898	5.8489
IHAAS	0.0119	0.0494	0.1426	2.4715
SNKRN	-8.4335 e-05	0.0408	-0.0052	3.9761
TRLIBOR	-0.0003	0.0089	-4.2401	36.1487

	JB	LB	LM
XUTUM	270.55* (*p* < *0.05*)	0.92 (*p* = *0.33*)	12.68 (*p* = *0.24*)
ASELS	17.53* (*p* = *0.00*)	0.00 (*p* = *0.98*)	33.69* (*p* = *0.00*)
CEOEM	8.66* (*p* = *0.01*)	5.68* (*p* = *0.01*)	46.41* (*p* < *0.05*)
FLAP	39.73* (*p* < *0.05*)	0,00 (*p* = *0.96*)	17.20 (*p* = *0.07*)
IHAAS	1.76 (*p* = *0.41*)	3.49 (*p* = *0.06*)	33.73* (*p* = *0.00*)
SNKRN	4.64 (*p* = *0.1*)	0,60 (*p* = *0.43)*	42.74* (*p* < *0.05*)
TRLIBOR	5707.4* (*p* < *0.05*)	1.55 (*p* = *0.21*)	4.43 (*p* = *0.92*)

Notes: In the table denotes that Std. Dev. and p are standart deviation and p-values, respectively. Jarque-Bera (JB) statistic shows that the Jarque-Bera test of normality statistics; where the null hypothesis (H_0) is defined as there is no difference between the distribution of the series and the normal distribution. Jarque-Bera (JB) statistic shows that the Ljung-Box (LB) test of autocorrelation statistics; where the null hypothesis (H_0) is defined as there is no autocorrelation in the series. ARCH-LM (LM) statistic shows that the ARCH-LM test of ARCH effects statistics; where the null hypothesis (H_0) is defined as there is no ARCH effects in the series. '*' indicate that null hypothesis (H_0) is rejected at 95% confidence level.

Table 3 shows parameter estimates of the GARCH-type models. The constant term of parameter ω, the $\psi 1$ parameter the effect of shocks/new news on the market on volatility, that is, the short-term conditional variance (ARCH term), the $\theta 1$ parameter represents the effect of the volatility of the previous period on the volatility of the next period, that is, the long-term conditional variance (GARCH term), the $\zeta1$ parameter indicates the effect of leverage on volatility, the δ parameter shows the power parameter, that is, the dependence of volatility on that period in the conditional variance equations of GARCH-type models. The constant term (ω) of the models is statistically significant at the 95% confidence level that CEOEM for GARCH-SN; ASELS for GARCH-ST, GJR-GARCH-N, GJR-GARCH-SN and GJR-GARCH-T; CEOEM for GARCH-SN, GJR-GARCH-N and GJR-GARCH-SN; SNKRN for GARCH-N, GARCH-SN, GARCH-ST and GJR-GARCH-N models, and the conditions of the models belonging to company are met. In models that meet the condition, If the model coefficients are to be examined in more detail, the ARCH effect parameter $\psi1$, which expresses the past shocks, is 0.5 approximately in SNKRN, while the GARCH effect parameter $\theta1$, which expresses the effect of the shocks in the previous period from the current period on the volatility of the next period, is in the range 0.44-0.54 in SNKRN. This indicates that approximately 50% of the SNKRN company's return consists of shocks from the past period, and approximately 44% from the shocks of the immediate previous period. Thus, it can be said that the volatility of SNKRN company is heavily affected by the shocks of the previous period. The $\zeta1$ parameter in the all companies for some models is positive. This parameter is positive shows that negative shocks affect volatility more than positive shocks. According to this result, this companies has a leverage effect of this period. The υ parameter that t distribution shape parameter in the all companies is in the range 3-4. The fact that this parameter is close to zero indicates that the shape of the distribution cannot be distorted, that is, it is not skewness or kurtosis. λ that the skewness term is aproximetly λ=1 for all companies. This parameter say that distribution is right-skewed.

Table 3. Model Parameters

Models	Parameters	Companies	
		ASELS	CEOEM
GARCH-N	ω	0.0000	0.0000
	p-value	0.1000	0.6269
	ψ_1	0.0000	0.0000
	p-value	0.9863	0.1000
	θ_1	0.9982*	0.9990*
	p-value	0.0000	0.0000
GARCH-SN	ω	0.0000	0.0000*
	p-value	1.0000	0.8758
	ψ_1	0.0000	0.0000
	p-value	0.9886	1.0000
	θ_1	0.9982*	0.9989*
	p-value	0.0000	0.0000
	λ	0.9599*	0.8532*
	p-value	0.0000	0.0000
GARCH-T	ω	0.0003	0.0005
	p-value	0.1161	0.0794
	ψ_1	0.4500	0.5389
	p-value	0.0846	0.0737
	θ_1	0.4286	0.3724*
	p-value	0.0572	0.0198
	υ	4.4764*	3.5236*
	p-value	0.0282	0.0039
GARCH-ST	ω	0.0003*	0.0005
	p-value	0.0479	0.0581
	ψ_1	0.5396	0.5418*
	p-value	0.0646	0.0174
	θ_1	0.3996*	0.3746*
	p-value	0.0268	0.0017
	λ	1.1825*	1.0098*
	p-value	0.0000	0.0000
	υ	3.9159*	3.5205*
	p-value	0.0178	0.0002
GJR-GARCH-N	ω	0.0000*	0.0006*
	p-value	0.0000	0.0036
	ψ_1	0.0000	0.2500

	p-value	0.9998	0.1168
	θ_1	1.0000*	0.4052*
	p-value	0.0000	0.0021
	ζ_1	-0.0160*	0.0805
	p-value	0.0000	0.6879
GJR-GARCH-SN	ω	0.0000*	0.0000*
	p-value	0.0000	0.0436
	ψ_1	0.0000	0.0000
	p-value	0.9998	0.9999
	θ_1	1.0000*	1.0000*
	p-value	0.0000	0.0000
	ζ_1	-0.0151*	-0.0088
	p-value	0.0000	0.4046
	λ	0.9770*	0.8460*
	p-value	0.0000	0.0000
GJR-GARCH-T	ω	0.0003*	0.0005
	p-value	0.0152	0.0757
	ψ_1	0.3302	0.3566
	p-value	0.1452	0.2249
	θ_1	0.4066*	0.3785*
	p-value	0.0336	0.0122
	ζ_1	0.2922	0.3250
	p-value	0.3997	0.4672
	υ	4.6185*	3.4286*
	p-value	0.0132	0.0030
GJR-GARCH-ST	ω	0.0003	0.0005
	p-value	0.0902	0.0801
	ψ_1	0.4153	0.3581
	p-value	0.2183	0.2248
	θ_1	0.3984	0.3811*
	p-value	0.0565	0.0113
	ζ_1	0.2216	0.3274
	p-value	0.6193	0.4648
	λ	1.1556*	1.0131*
	p-value	0.0000	0.0000
	υ	4.1529*	3.4246*
	p-value	0.0287	0.0029
Models	Parameters	Companies	

		IHAAS	SNKRN
GARCH-N	ω	0.0002	0.0002*
	p-value	0.2391	0.0219
	ψ_1	0.2604*	0.4763*
	p-value	0.0160	0.0151
	θ_1	0.6753*	0.4527*
	p-value	0.0000	0.0000
GARCH-SN	ω	0.0001	0.0002*
	p-value	0.2832	0.0339
	ψ_1	0.3136*	0.4676*
	p-value	0.0108	0.0128
	θ_1	0.6736*	0.4911*
	p-value	0.0000	0.0000
	λ	1.3520*	1.1354*
	p-value	0.0000	0.0000
GARCH-T	ω	0.0002	0.0002
	p-value	0.2901	0.0403
	ψ_1	0.2682*	0.5528*
	p-value	0.0237	0.0037
	θ_1	0.6769*	0.4462*
	p-value	0.0000	0.0000
	υ	99.3423	4.4461*
	p-value	0.8531	0.0037
GARCH-ST	ω	0.0001	0.0002*
	p-value	0.3147	0.0356
	ψ_1	0.3367*	0.5362*
	p-value	0.0185	0.0021
	θ_1	0.6623*	0.4628*
	p-value	0.0000	0.0000
	λ	1.3851*	1.0610*
	p-value	0.0000	0.0000
	υ	17.4357	4.6139*
	p-value	0.4975	0.0039
GJR-GARCH-N	ω	0.0002	0.0002*
	p-value	0.2004	0.0245
	ψ_1	0.2754*	0.5370*
	p-value	0.0238	0.0270
	θ_1	0.6907*	0.4829*

	p-value	0.0000	0.0000
	ζ_1	-0.1034	-0.1907
	p-value	0.5990	0.5181
GJR-GARCH-SN	ω	0.0001	0.0001*
	p-value	0.2253	0.0376
	ψ_1	0.3346*	0.5402*
	p-value	0.0178	0.0203
	θ_1	0.6922*	0.5450*
	p-value	0.0000	0.0000
	ζ_1	-0.1234	-0.2561
	p-value	0.5252	0.3681
	λ	1.3606*	1.1586*
	p-value	0.0000	0.0000
GJR-GARCH-T	ω	0.0002	0.0002
	p-value	0.5012	0.0585
	ψ_1	0.2807*	0.7019*
	p-value	0.0168	0.0126
	θ_1	0.6899*	0.4955*
	p-value	0.0000	0.0000
	ζ_1	0.7470	-0.3970
	p-value	-0.0920	0.2619
	υ	97.8754	4.2144*
	p-value	0.7408	0.0011
GJR-GARCH-ST	ω	0.0001	0.0002
	p-value	0.6341	0.0643
	ψ_1	0.3659	0.6983*
	p-value	0.1227	0.0101
	θ_1	0.6744*	0.5379*
	p-value	0.0003	0.0000
	ζ_1	-0.0759	-0.4535
	p-value	0.8678	0.2216
	λ	1.3979*	1.1043*
	p-value	0.0000	0.0000
	υ	18.0498	4.3232*
	p-value	0.7331	0.0013

[2]Note: Parameters marked with '*' indicate parameters that are significant at the 95% confidence level.

Table 4 and 5 shows values of information criteria of models of agriculture, forestry, fishing and hunting companies. This information criterias that Mean Absolute Error (MAE) and Mean Squared Error (MSE) as follows:

$$MAE = \frac{1}{N}\sum_{i=1}^{N}\left|\widehat{Y_\iota} - Y_i\right| \tag{34}$$

$$MSE = \frac{1}{N}\sum_{i=1}^{N}(\widehat{Y_\iota} - Y_i)^2 \tag{35}$$

Here, N is observations (N=2509).

According to the values given in Table 4 and 5, the best modeling is GARCH-T for ASELS, GJR-GARCH-N for CEOEM, IHASS and SNKRN. So, it was concluded that the GJR-GARCH-N model that best models time-varying beta risk differs according to companies.

Table 4. Estimation performance with MAE ($x10^2$) criteria of Time-varying Beta Risk with GARCH-type models

Companies/ Models	ASELS	CEOEM	IHAAS	SNKRN
GARCH-N	0.1433	0.2555	0.2135	0.1777
GARCH-SN	0.1434	0.2496	0.2149	0.1797
GARCH-T	0.1352*	0.2355	0.2139	0.1847
GARCH-ST	0.1464	0.2362	0.2159	0.1841
GJR-GARCH-N	0.1446	0.2143*	0.2098*	0.1720*
GJR-GARCH-SN	0.1446	0.2509	0.2109	0.1728
GJR-GARCH-T	0.1376	0.2397	0.2106	0.1795
GJR-GARCH-ST	0.1437	0.2408	0.2157	0.1793
[3]Note: '*' means that the model with the smallest value fits the data better.				

Table 5. Estimation performance with MSE ($x10^4$) criteria of Time-varying Beta Risk with GARCH-type models

Companies/ Models	ASELS	CEOEM	IHAAS	SNKRN
GARCH-N	0.0588	0.1188	0.0823	0.0748
GARCH-SN	0.0587	0.1160	0.0855	0.0764
GARCH-T	0.0555*	0.1171	0.0828	0.0804
GARCH-ST	0.0608	0.1179	0.0869	0.0802
GJR-GARCH-N	0.0585	0.1022*	0.0792*	0.0717*
GJR-GARCH-SN	0.0585	0.1153	0.0818	0.0722
GJR-GARCH-T	0.0591	0.1280	0.0799	0.0780
GJR-GARCH-ST	0.0622	0.1296	0.0860	0.0777
[3]Note: '*' means that the model with the smallest value fits the data better.				

Table 4 gives descriptive statistics of time-varying beta risks of companies. When the beta parameter is accepted as a risk measure, it can be said that the model with the highest volatility belongs to the ASELS company, which varies in the range of [0.1275;1.9718]. It can be said that investments with beta risk less than 1 that have lower risk than XUTUM investment while investments with beta risk more than that have higher risk than XUTUM investment. The negative beta risks indicate that the companies are in the opposed direction with the market. The average beta risk of less than 1 indicates that companies are less sensitive to the market while the average beta risk of greater than 1 indicates that companies are highly sensitive to the market. Thus, it is concluded that the sensitivity of the sector to the market is mean.

Table 6. Descriptive Statistics of Time-varying Beta Risk

Companies/ Beta risk	ASELS	CEOEM	IHAAS	SNKRN
Min.	0.1275	0.4079	0.6589	0.4342
Max.	1.9718	1.0182	1.6447	1.0840
Median	1.3800	0.6566	1.0605	0.6990
Mean	1.2559	0.6414	1.0361	0.6829
Std. Dev.	0.5154	0.1195	0.1930	0.1272
Skewness	-0.6792	0.1309	0.1309	0.1309
Kurtosis	2.3711	2.8538	2.8538	2.8538

Figure 2 shows the time series graphs of time-varying beta risks of the model that best models of these companies.

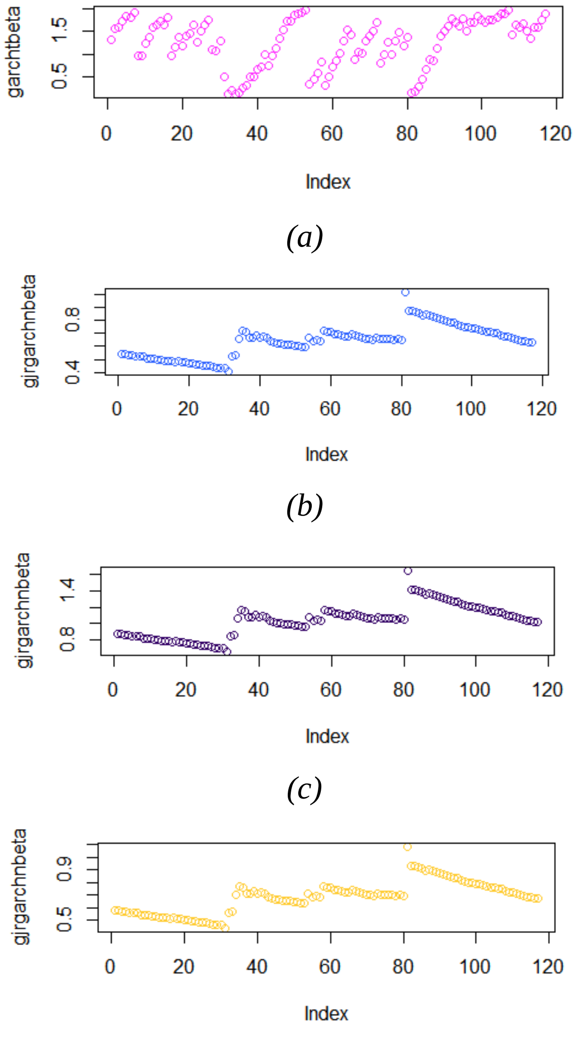


Figure 2. Time Series Plots of Beta Risk: (a) ASELS; (b) CEOEM; (c) IHAAS; (d) SNKRN

4. DISCUSSION AND CONCLUSION

The research data of this paper covers the dates of 04.11.2021-18.04.2022. In this date range, a portfolio was created by taking daily frequency data of all the information, communications, security, investigative activities, defense, office management, office support and other company support activities companies in the BIST National All index. This paper was conducted for the beta risk or systematic risk, that is the investors who create the risk cannot avoid, for the first time BIST National All index and all companies belonging to the information, communications, security, investigative activities, defense, office management, office support and other company support activities companies are used the daily frequency data on the date of last ten years which 4 November 2011 to 18 April 2021. For the time-varying beta risk parameters, the Conditional Capital Asset Pricing Model (C-CAPM) is used. Time-varying Linear Market Model (Tv-LMM) that is a data production model consistent with C-CAPM is modeled with GARCH-N, GARCH-SN, GARCH-T, GARCH-ST, GJR-GARCH-N, GJR-GARCH-SN, GJR-GARCH-T and GJR-GARCH-ST that are univariate GARCH-type models. In this paper, three main conclusions were reached and contributed to the practice literature. Firstly, according to the model benchmarking criterias for GARCH-type model which best models the time-varying beta risk; The best modeling is GARCH-T for ASELS, GJR-GARCH-N for CEOEM, IHASS and SNKRN. So, it was concluded that the GJR-GARCH-N model that best models time-varying beta risk differs according to companies. Secondly, the date 2020 was defined as the COVID-19 global epidemic by the WHO was March 11, 2020, the date of the 59th presidential election in the USA was December 12, 2020, the economic crisis observed in the Turkish economy in 2018, and the effects on the markets consequently of the global economic crisis experienced in 2008-2012 effect, that is extreme fluctuations, was clearly observed and there is a leverage effect in all companies on this period. Thirdly, $\zeta 1$ parameter in the all companies for some models is positive. This parameter is positive shows that negative shocks affect volatility more than positive shocks. Fourthly, It is concluded that the sensitivity of the sector to the market is mean. Finally, invest in XUTUM during the research period will make a profit financially. In future studies, it is recommended to compare the performance of models in different financial markets, periods and frequencies and to create investment portfolios.

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